Math 20550 - Calculus III - Summer 2014
July 16, 2014
Exam 3

Name: ____

There is no need to use calculators on this exam. This exam consists of 11 problems on 11 pages. You have 75 minutes to work on the exam. There are a total of 105 available points and a perfect score on the exam is 100 points. All electronic devices should be turned off and put away. The only things you are allowed to have are: a writing utensil(s) (pencil preferred), an eraser, and an exam. No notes, books, or any other kind of aid are allowed (except your notecard). All answers should be given as exact, closed form numbers as opposed to decimal approximations (i.e., π as opposed to 3.14159265358979...). **You must show all of your work to receive credit. Please box your final answers.** Cheating is strictly forbidden. Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in, nor tolorate academic dishonesty. My signature here binds me to the Notre Dame Honor Code:

Signature:____

Problem	Score
1	/10
2	/10
3	/10
4	/10
5	/10
6	/10
7	/10
8	/10
9	/10
10	/5
11	/10
Score	/100

Problem 1 (10 points)**.** *Compute the double integral*

$$\int_0^{\frac{\pi}{2}} \int_y^{\frac{\pi}{2}} \sin(x^2) dx dy$$

and sketch the region of integration.

Problem 2 (10 points)**.**

(a - 7 points) *Combine*

$$\int_0^2 \int_0^x \sqrt{x^2+y^2}dydx + \int_2^{2\sqrt{2}} \int_0^{\sqrt{8-x^2}} \sqrt{x^2+y^2}dydx$$

into a single computable *integral. (*Hint*: sketch the region of integration)*

(b - 3 points) *Compute the integral in* (*a*)*.*

Problem 3 (10 points)**.** *Rewrite the triple integral*

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} xy\, dzdydx$$

using spherical coordinates. You do not need to compute it.

Problem 4 (10 points)**.** *A thin spring has the shape of the helix*

$$x = t, y = \cos t, z = \sin t, 0 \leq t \leq 6\pi$$

and has linear density function $\rho(x, y, z) = x^2 + y^2 + z^2$. *Find the mass of the spring.*

Problem 5 (10 points)**.** *Compute* $\int_C \mathbf{F} \cdot d\mathbf{r}$ *where* $\mathbf{F} = (y^2z + 2xz^2)\mathbf{i} + 2xyz\mathbf{j} + (xy^2 + 2x^2z)\mathbf{k}$ *and* C *has parametric equations* $x = \sqrt{t}$, $y = t + 1$, *and* $z = t^2$, $0 \leq t \leq 1$.

Problem 6 (10 points). *Consider the integral* $\iint_R \frac{x-2y}{3x-y}dA$*, where* R *is the parallelogram enclosed by the lines* $x-2y=0$*,* $x-2y=4$*,* $3x-y=1$*, and* $3x-y=8$*. Use a change of coordinates to rewrite the integral as a double integral over a rectangular region* S*. Sketch the region* S*. You do not have to compute the integral.*

Problem 7 (10 points)**.** *Compute the area inside the ellipse* $\dfrac{x^2}{4} + y^2 = 1$ *using an appropriate type of integral.*

Problem 8 (10 points)**.** *Find the volume enclosed by the cones* $z = \sqrt{x^2 + y^2}$ *and* $2 - z = \sqrt{x^2 + y^2}$.

Problem 9 (10 points)**.** *Compute the divergence and curl of the vector field*

$$\mathbf{F} = xye^z\mathbf{i} + yze^x\mathbf{k}.$$

Problem 10 (5 points)**.** *Suppose that* $\mathbf{F}$ *is a conservative vector field and that* $\mathbf{F}$ *is* C^1*. Compute the curl of* $\mathbf{F}$*.*

Problem 11 (10 points). *Compute* $\int_C (y\cos x - xy\sin x)dx + (xy + x\cos x)dy$ *where* C *is the triangle traced out in moving from* $(0,0)$ *to* $(0,4)$ *to* $(2,0)$ *and back to* $(0,0)$.